CHAPTER SIX

LINEAR SEQUENCE

Sequence:

-A sequence is a set of numbers or terms written in a definite order, with a rule or formula for obtaining the terms.

Example(1)
1, 3, 5, 7, 9, 11……………..
- In this given sequence, each term is obtained by adding 2 to the preceding term or the previous term.

Example(2):
1, 3, 9, 27…
In this given sequence, each term is obtained by multiplying the previous term by 3.

Series:

- When all the terms of a sequence are added together, we get what we call a series.
- For example, consider the sequence 2, 4, 6, 8, 10 ……..
- If the terms of this series are added together, we shall get 2 + 4 + 6 + 8 + 10 ……
- This is what is referred to as a series.
- Since such a series does not contain a fixed number of terms, it is referred to as an infinite series.
- Consider the sequence 2, 4, 8 …….32.
- By adding the terms together, we get the series 2 + 4 + 8 …… + 32.
- Since this series contains a definite number of terms, it is referred to as a finite series.

Types of sequence:

- Two types of sequence shall be considered and these are:

(i) Arithmetic progression (AP), which is also known as linear sequence.

(b) Geometric progression (G.P), which is also known as exponential sequence.

Arithmetic progression:

- In this type, any term differs from the preceding term by a constant known as the common difference.
- This common difference which is represented by d, can either be positive or negative.

  Examples(1) 2, 5, 8, 11 ……………
- In this, the common difference = 5 – 2 = 3 or 8 – 5 = 3.

  Example(2) 3, 13, 23 ……..
- In this, the common difference = 13 – 3 = 10 or 23 – 13 = 10.

  Example (3) -2, - 4, - 6, -8 ……………………
- In this case, d = - 6 – (- 4) = - 6 + 4 = -2.

  - The first term of an A.P is represented by U_1, the second term byU_2 and the third by U_3 and so on.

  - With reference to A.P, the following must be taken notice of:

  U_1= 1st term = a

  U_2 = 2nd term = a + d

  U_3= 3rd term = a + 2d

  U_4= 4th term = a + 3d

  U_n = nth term = a + (n-1)d
- The nth term of an A.P is therefore given by $U_n = a+(n-1)d$.
- This is the formula for finding any term of a linear sequence.

(Q1) Find the 11^{th} term of a linear sequence of the form 4, 9, 14, 19 …………

Soln:

$a = 4$ and $d = 9 – 4 = 5$.

From $U_n = a + (n -1)d$

$\Rightarrow U_{11} = a + (11 – 1)d$

$\Rightarrow U_{11} = 4 + (11 – 1) \times 5$

$\Rightarrow U_{11} = 4+5(10) = 4 + 50$

$= 54$.

(Q2) Find the 20^{th} term of the sequence 4, 6, 8, 10, 12 ……

Soln:

$a = 4$ and $d = 6 - 4 = 2$.

Since $U_n = a + (n -1)d$, then

$U_{20} = 4 + (20 – 1) 2$

$U_{20} = 4 + (19) \times 2$

$\Rightarrow U_{20} = 4 + 2(19)$

$\Rightarrow U_{20} = 4 + 38$

$\Rightarrow U_{20} = 42$.

(Q3) Find the 8^{th} term of a linear sequence of the form 47, 42, 37, 32 ………

Soln:

$a = 47$ and $d = 42 – 47 = - 5$.

Since $U_n = a + (n - 1)d$,

=> $U_8 = 47 + (8 – 1) (-5)$

=> $U_8 = 47 + (-5)(8 - 1)$

=> $U_8 = 47 - 5(7)$

=> $U_8 = 47 - 35 = 12$

(Q4) Show that the 90^{th} term of the sequence 2, 7, 12, 17 ………… is 447.

Soln:

a = 2 and d = 7 – 2 = 5.

From $U_n = a + (n - 1)d$

=> $U_{90} = 2 + (90 – 1) 5$

=> $U_{90} = 2 + 5(89)$

=> $U_{90} = 2 + 445 = 447$.

(Q5) Find the 12^{th} term of an A.P of the form 7, $6\frac{1}{4}, 5\frac{1}{2} \ldots \ldots \ldots \ldots \ldots \ldots ..$

Soln:

a = 7, and d = $6\frac{1}{4} - 7 = 6.25 – 7 = - 0.75$.

Since $U_n = a + (n - 1)d$

=> $U_{12} = 7 + (12 - 1)(-0.75)$

=> $U_{12} = 7 + (11)(-0.75)$

=> $U_{12} = 7 + (-8.25)$

=> $U_{12} = 7 – 8.25 = -1.25$.

(Q6) Find the linear sequence whose 8^{th} term is 38 and 22^{nd} term is 108.

Soln:

$U_n = a + (n - 1)d$.

The 8th term =

$U_8 = a + (8 - 1)d$

=> $U_8 = a+7d$

Since the 8th term = 38

=> a + 7d = 38 …………… eqn (1).

Also the 22nd term = 108.

$U_{22} = a + (22-1)d$

=> $U_{22} = a + 21d$.

Since the 22nd term = 108.

=> a + 21d = 108 ………………. eqn (2)

Solving eqn (1) and eqn (2) simultaneously => a = 3 and d = 5.

The sequence = a, a + d, a + 2d, a + 3d …………………..

= 3, 3+5, 3 + 2(5), + 3 + 3(5) …………….

= 3, 8, 13, 18 ……………………..

(Q7 The fourth term of a linear sequence is 19 and the eleventh term is 54.Find the 8th term.

Soln:

$U_n = a + (n - 1)d$

The fourth term =

$U_4 = a + (4 - 1)d$

=> $U_4 = a + 3d$.

But the fourth term = 19

=> a + 3d = 19 ……………. eqn (1)

The 11th term = 54.

$U_{11} = a + (11 – 1)d$

=> $U_{11} = a + 10d$.

Since the 11th term = 54

=> a + 10d = 54 …………. eqn (2)

Solving eqn (1) and eqn (2) simultaneously => d = 5 and a = 4.

The 8th term of the sequence is given by $U_8 = a + (8-1) d$

=> $U_8 = a + 7d$ => $U_8 = 4 + 7(5) = 4 + 35 = 39$.